1. (20%) Assume that a message signal is given by

$$m(t) = 2\cos(2\pi f_m t) + \cos(4\pi f_m t)$$

Calculate an expression for

$$X_c(t) = \frac{1}{2} A_c m(t) \cos(2\pi f_c t) \pm \frac{1}{2} A_c \hat{m}(t) \sin(2\pi f_c t)$$

for $A_c=4$. Show that the result is upper-sideband or lower-sideband SSB depending upon the choice of the algebraic sign.

2. An FM modulator has output

$$X_c(t) = 100 \cos \left[2\pi f_c t + 2\pi f_d \int_0^t m(\alpha) d\alpha \right]$$

where $f_d = 20 \, \text{Hz/V}$. Assume that m(t) is the rectangular pulse $m(t) = 4 \Pi \left[\frac{(t-4)}{8} \right]$.

- (a) (6%) Sketch the phase deviation in radians.
- (b) (6%) Sketch the frequency deviation in hertz.
- (c) (6%) Determine the peak frequency deviation in hertz.
- (d) (6%) Determine the peak phase deviation in radians.
- (e) (6%) Determine the power at the modulator output.
- 3. (20%) Let X(t)=Asin(Wt+Y), where A, W and Y are independent random variables. Assume A has mean 9 and variance 25, Y is uniform on [-2π,2π], and W is uniform on [-20, 20]. Find the mean and autocorrelation for the random process X(t).
- 4. Suppose the impulse response of a linear system is given by h(t)=3, 2< t<5.
 - (a) (10%) If the input X(t) to this system is a white noise with spectral density No/2=5, find the autocorrelation function Ry(τ), where Y(t) is the output.
 - (b) (10%) Now suppose the signal $s(t)=b_0P(t)+b_1P(t-3)+b_2P(t-6)$, where P(t)=1, 0 < t < 3, and $b_i=1$ or -1, is applied to the input of the system. For the case $b_0=b_2=-1$ and $b_1=1$, sketch the resulting output.
 - (c) (10%) Now suppose the signal s(t)+X(t) is applied to the input of the system. At what 3 instants should the output be sampled to detect bo, b1 and b2? What is the variance of the output statistic at these sampling instants?