招生學年度	103	招生類別	碩士班
系所班別	電機光電碩士班聯合招生（電機工程學系碩士班、電機工程學系 電子工程碩士班、光電工程學系碩士班）		
科目名稱	線性代數		
注意事項	本考科可使用掌上型計算機		

1. Solve the following equations by using x_3 and x_4 as the free variables α and β.
$$x_1 + 3x_2 + x_3 + x_4 = 3$$
$$2x_1 - 2x_2 + x_3 + 2x_4 = 8 \quad (10\%)$$
$$x_1 - 5x_2 + x_4 = 5$$

2. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & -1 & -3 \\ 2 & 3 & 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}$. If $A\vec{x} = \vec{b}$, find $\vec{x}$. (10%)

3. Find the determinant of $\begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \\ 6 & -3 & 4 \end{bmatrix}$. (10%)

4. Let $A = \begin{bmatrix} 1 & 4 & 3 \\ -1 & -2 & 0 \\ 2 & 2 & 3 \end{bmatrix}$, find A^{-1}. (10%)

5. $\vec{a} = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}, \vec{b} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \vec{c} = \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix}$.

   Are $\vec{a}, \vec{b}, \vec{c}$ linearly dependent? Why? (10%)

6. $A = \begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix}$. Find the eigenvalues and the corresponding eigenvectors of A. (10%)

7. Let $\mathbf{u}_1 = (1, 1, 1)^T$, $\mathbf{u}_2 = (1, 2, 2)^T$, $\mathbf{u}_3 = (2, 3, 4)^T$
   $\mathbf{v}_1 = (4, 6, 7)^T$, $\mathbf{v}_2 = (0, 1, 1)^T$, $\mathbf{v}_3 = (0, 1, 2)^T$.
   (a) Find the transition matrix from $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ to $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$. (10%)
   (b) If $\mathbf{x} = 2\mathbf{v}_1 + 3\mathbf{v}_2 - 4\mathbf{v}_3$, find the coordinates of $\mathbf{x}$ with respect to $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$. (10%)

8. Find the best least squares fit by a linear function to the data below: (20%)

x	−1	0	1	2
y	0	1	3	9